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Containment control of leader-following multi-agent systems with Markovian switching network topologies and measurement noises ☆

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ABSTRACT

This paper investigates the distributed containment tracking control problem for first-order agents with multiple dynamic leaders under directed Markovian switching network topologies. The control input of each agent can only use its local state and the states of its neighbors corrupted by white noises. Firstly, when the dynamic leaders' velocities have some special forms, which include the stationary leaders as a special case, some necessary and sufficient conditions are presented for the containment tracking in the asymptotic unbiased mean square sense. Secondly, for more general leaders' velocities, necessary and sufficient conditions are provided for the containment tracking with bounded errors in the mean square sense. Finally, a simulation example is given to illustrate the performance of the proposed control scheme.

1. Introduction

Containment control is concerned with multi-agent systems with multiple leaders and single or multiple followers, aiming to design appropriate control protocols to drive the follower(s) to the convex hull of the leaders asymptotically or in finite time. It has attracted a lot of interest recently (Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Lou & Hong, 2012; Meng, Ren, & You, 2010; Notarstefano, Egerstedt, & Haque, 2011; Tang, Huang, & Shao, 2012). Distributed containment control of first-order discrete multi-agent systems with multiple stationary leaders and noisy measurements is studied in Tang et al. (2012) whereas Lou and Hong (2012) consider a second-order multi-agent system with random switching topologies. In Ji et al. (2008), the problem of driving a set of mobile robots to a given target destination is studied. Distributed finite-time attitude containment control for multiple rigid bodies is addressed in Meng et al. (2010) and Notarstefano et al. (2011) investigate a first-order network model in which stationary leaders (the female moths) and moving followers (the males) are only intermittently visible by each other.

To our knowledge, there is no existing result on containment control of multi-agent systems under both Markovian switching topologies and channel noises. Inspired by the work (Li & Wu, 2013), this paper studies the containment tracking control for first-order agents with a target set specified by multiple leaders under Markovian switching topologies. The control input of each agent can only depend on its local state and the states of its neighbors corrupted by stochastic communication noises. The preliminary version of the paper was presented at the 32nd Chinese Control Conference (Li, Xie, & Zhang, 2013). Comparing with the existing work, the contributions of the paper include:

(1) Different from the existing results such as those in Lou and Hong (2012), this paper not only solves the containment tracking problem, but also points out the specific points that the followers will eventually converge to.
(2) This paper is the first to study the containment control with both Markovian switching topologies and stochastic noises in communication channels.

2. Preliminaries

Let $r(t)$ be a right-continuous homogeneous Markov process on the probability space taking values in a finite state space

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$S = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{pq})_{N\times N}$ given by

$$P_{pq}(t) = P\{r(t+s) = q|r(s) = p\} = \begin{cases} \gamma_{pq}t + o(t) & \text{if } p \neq q \\ 1+\gamma_{pp}t + o(t) & \text{if } p = q \end{cases} \quad (1)$$

for any $s, t \geq 0$. Here $\gamma_{pq} > 0$ is the transition rate from p to q if $p \neq q$ while $\gamma_{pp} = -\sum_{q=1,q\neq p}^{N} \gamma_{pq}$. Now, we can define a time-varying digraph $\mathcal{G}_{r(t)} = (\mathcal{V}, \mathcal{E}(r(t)), A(r(t)))$ of order n with the set of nodes $\mathcal{V} = \{1, 2, \ldots, n\}$, set of arcs $\mathcal{E}(r(t)) \subset \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A(r(t)) = (a_{ij}(r(t)))_{n\times n}$ with nonnegative elements. $(j, i) \in \mathcal{E}(r(t))$ means that agent j can directly send information to agent i; the set of neighbors of vertex i is denoted by $\mathcal{N}_i(r(t)) = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}(r(t)),\ i \neq j\}$. $a_{ij}(r(t)) > 0$ when $j \in \mathcal{N}_i(r(t))$ and $a_{ij}(r(t)) = 0$ otherwise. A sequence $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ of edges is called a directed path from node i_1 to node i_k. A digraph is strongly connected if there exists a path between any two distinct nodes. The diagonal matrix $D(r(t)) = \mathrm{diag}(d_1(r(t)), d_2(r(t)), \ldots, d_n(r(t)))$ is the degree matrix with diagonal elements $d_i(r(t)) = \sum_{j\in\mathcal{N}_i} a_{ij}(r(t))$. The Laplacian of a weighted digraph $\mathcal{G}_{r(t)}$ is defined as $L(r(t)) = D(r(t)) - A(r(t))$.

We consider a system consisting of n followers and m leaders which is depicted by a graph $\bar{\mathcal{G}}_{r(t)} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}(r(t)))$, where $\bar{\mathcal{V}} = \mathcal{V}_f \cup \mathcal{V}_l$, $\mathcal{V}_f = \{1, 2, \ldots, n\}$, $\mathcal{V}_l = \{n+1, n+2, \ldots, n+m\}$, and its set of arcs $\bar{\mathcal{E}}(r(t)) \subset \left(\mathcal{V}_f \times \mathcal{V}_f \cup \mathcal{V}_f \times \mathcal{V}_l\right)$. If for every node i in $\mathcal{V}_f$, one can find a node j in $\mathcal{V}_l$, such that there is a path in $\bar{\mathcal{G}}_{r(t)}$ from node j to i, we say that the set $\mathcal{V}_l$ is globally reachable in $\bar{\mathcal{G}}_{r(t)}$. A diagonal matrix $B(r(t)) = \mathrm{diag}(\sum_{s=1}^{m} b_{1s}(r(t)), \ldots, \sum_{s=1}^{m} b_{ns}(r(t)))$ is the leader adjacency matrix associated with $\bar{\mathcal{G}}_{r(t)}$, where $b_{is}(r(t)) > 0$ if node s in $\mathcal{V}_l$ is a neighbor of node i in $\mathcal{V}_f$ and $b_{is}(r(t)) = 0$ otherwise. The union graph of a graph series $\bar{\mathcal{G}}_1, \ldots, \bar{\mathcal{G}}_N$, with the same node set $\bar{\mathcal{V}}$ for some $N \geq 1$, is defined as the graph $\bigcup_{i=1}^{N} \bar{\mathcal{G}}_i$, whose node set is $\bar{\mathcal{V}}$ and edge set is the union of the edge sets of all the collection graphs, and the connected weight between agent i and j is the sum weight $\sum_{p=1}^{N} a_{ij}(p)$, and the connected weight between the follower i and the leader set is the sum of $\sum_{p=1}^{N} b_{is}(p)$, $s \in \mathcal{V}_l$.

3. Problem formulation

Consider the following multi-agent system:

$$\dot{x}_i = u_i, \quad i = 1, \ldots, n, \quad (2)$$

where $x_i \in R$, and $u_i \in R$ are the position, and the control input of agent i, respectively.

The leaders for (2) are described by

$$\dot{h}_i = f_i(h, t), \quad i = n+1, \ldots, n+m, \quad (3)$$

where $h_i \in R$, and $f_i(h, t) \in R$ are the ith leader's position and velocity, respectively, $h = (h_{n+1}, \ldots, h_{n+m})^T$.

Remark 1. In (2), the derivative of x_i is in fact the control input (can be viewed as the velocity). This first order integrator model has been widely adopted in the literature when consensus is to be achieved for the positions of agents. In practice, the consensus of the first order integrator systems serves as generating reference trajectories for agents (e.g. way points of unmanned vehicles).

In this paper, we consider the leader-following system (2)–(3) with Markovian switching graph $\bar{\mathcal{G}}_{r(t)}$. The Markov process $r(t)$ is defined in (1). We also assume that the Markov process $r(t)$ is ergodic. This means that the Markov process admits a unique stationary distribution $\pi = (\pi_1, \ldots, \pi_N)$. Therefore, we assume that the graph starts as a stationary one.

In the graph $\bar{\mathcal{G}}_{r(t)}$, the ith agent can receive information from other agents:

$$x_{ij}^{f}(r(t)) = \begin{cases} x_j + \sigma_{ij}(r(t))\xi_{ij}, & j \in N_i(r(t)), \\ 0, & j \notin N_i(r(t)), \end{cases}$$

$$x_{is}^{l}(r(t)) = \begin{cases} h_s + \sigma_{is}(r(t))\xi_{is}, & s \in N_i(r(t)), \\ 0, & s \notin N_i(r(t)), \end{cases}$$

where $\{\xi_{ij}, i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n,\ s = n+1, \ldots, n+m\}$ are independently standard white noises and $\sigma_{ij}(r(t)) \geq 0$ and $\sigma_{is}(r(t)) \geq 0$ are the noise intensities.

We use the following distributed switching control law:

$$u_i = -a(t)\rho(r)\sum_{j=1}^{n} a_{ij}(r(t))(x_i - x_{ij}^{f}(r(t))) - a(t)\rho(r)\sum_{s=n+1}^{n+m} b_{is}(r(t))(x_i - x_{is}^{l}(r(t))), \quad (4)$$

where $\rho(r) = 1/\sum_{l=1}^{N} \pi_l \chi_{r(t)=l}$, $a(t) : [0, \infty) \to (0, \infty)$ is a piecewise continuous function to be designed later.

Define

$$B_l(r) = \begin{bmatrix} b_{1,n+1}(r) & \cdots & b_{1,n+m}(r) \\ \vdots & \ddots & \vdots \\ b_{n,n+1}(r) & \cdots & b_{n,n+m}(r) \end{bmatrix},$$

and

$$B(r) = \mathrm{diag}\Big(\sum_{i=n+1}^{n+m} b_{1i}(r), \ldots, \sum_{i=n+1}^{n+m} b_{n,i}(r)\Big),$$

$$\Sigma_{1i} = (a_{i1}(r)\sigma_{i1}(r), \ldots, b_{i,n+m}(r)\sigma_{i,n+m}(r)),$$

$$G(r) = \mathrm{diag}\big(\Sigma_{11}, \ldots, \Sigma_{1n}\big), \qquad H(r) = L(r) + B(r),$$

substituting (4) into (2) yields

$$dx = -a(t)\rho(r)H(r)xdt + a(t)\rho(r)B_l(r)hdt + a(t)\rho(r)G(r)d\omega, \quad (5)$$

where $x = (x_1, \ldots, x_n)^T$ and $\omega = (\omega_{11}, \ldots, \omega_{n1}, \ldots, \omega_{n,n+m})^T$ is an $n(n+m)$-dimensional standard Brownian motion. We assume that the Markov process $r(t)$ is independent of the Brownian motion $\omega(t)$.

Now, we give the definitions of containment for system (2)–(3), which describes the dynamics of the tracking error as $t \to \infty$.

Definition 1. The containment can be solved in the asymptotic unbiased mean square sense if for any $i \in \mathcal{V}_f$, there exist nonnegative constants $\kappa_{ij} (j = n+1, \ldots, n+m)$ satisfying $\sum_{j=n+1}^{n+m} \kappa_{ij} = 1$ such that $\lim_{t\to\infty} E|x_i - \sum_{j=n+1}^{n+m} \kappa_{ij}h_j|^2 = 0$.

Definition 2. The containment with bounded error can be solved in the mean square sense if $\exists M > 0$, for any $i \in \mathcal{V}_f$, there exist nonnegative constants $\kappa_{ij} (j = n+1, \ldots, n+m)$ satisfying $\sum_{j=n+1}^{n+m} \kappa_{ij} = 1$ such that $\limsup_{t\to\infty} E|x_i - \sum_{j=n+1}^{n+m} \kappa_{ij}h_j|^2 \leq M$.

Defining $H = \sum_{i=1}^{N} H(i)$, $B = \sum_{i=1}^{N} B(i)$ and $B_l = \sum_{i=1}^{N} B_l(i)$, we can get the following two lemmas, which will be useful in deriving the main results of this paper.

Lemma 1. *H is positive stable if and only if the leader set $\mathcal{V}_l$ is globally reachable in the union digraph $\bigcup_{i=1}^{N} \bar{\mathcal{G}}_i$.*

Proof. The digraph composed of n followers is described by $\mathcal{G}_i$, whose union digraph is given by $\bigcup_{i=1}^{N} \mathcal{G}_i$. The corresponding Laplacian matrix for $\bigcup_{i=1}^{N} \mathcal{G}_i$ is $L = \sum_{i=1}^{N} L(i)$.

Necessity: If the leader set $\mathcal{V}_l$ is not globally reachable in the union digraph $\bigcup_{i=1}^{N}\bar{\mathcal{G}}_i$, we show that H is not positive stable in the following. For brevity, we only prove the case when $\bigcup_{i=1}^{N}\mathcal{G}_i$ is strongly connected. Other cases follow in a similar manner.

In this case, since the leader set is not globally reachable, one gets $B = 0$. Noting that $H = L + B = L$, H has zero eigenvalues and therefore is not positive stable.

Sufficiency: The proof of sufficiency is similar to Lemma 3.5 in Tang et al. (2012).

Lemma 2. *If the leader set $\mathcal{V}_l$ is globally reachable in the union digraph $\bigcup_{i=1}^{N}\bar{\mathcal{G}}_i$, then the ith element of $h_c = H^{-1}B_l h$ can be written as $h_{ci} = \sum_{j=n+1}^{n+m} k_{ij}h_j$ with the nonnegative constant k_{ij} satisfying $\sum_{j=n+1}^{n+m} k_{ij} = 1,\ i = 1, \ldots, n$.*

Proof. Since the leader set $\mathcal{V}_l$ is globally reachable in the union digraph $\bigcup_{i=1}^{N}\bar{\mathcal{G}}_i$, by Lemma 1, H is positive stable, which, by the definition of H, implies that H is an M-matrix. By the properties of M-matrix, H^{-1} is a nonnegative matrix.

From the definition of H, one has $H\mathbf{1}_n = (L+B)\mathbf{1}_n = B_l\mathbf{1}_n$, which yields $H^{-1}B_l\mathbf{1}_n = \mathbf{1}_n$. By noting that H^{-1} is a nonnegative matrix, one can get the conclusion.

Remark 2. When the leader set $\mathcal{V}_l$ only contains one leader, Lemma 1 will reduce to Lemma 4 in Hu and Hong (2007). Therefore, the result in Hu and Hong (2007) for single leader case can be considered as a special case of Lemma 1.

In Tang et al. (2012), the leaders set being globally reachable in a fixed topology is only a sufficient condition to guarantee that H is positive stable. In Lemma 1, for the Markovian switching topologies, we give a sufficient and necessary condition for H to be positive stable.

Remark 3. Some comments on the Markovian graph model are given below:

(1) About the Markovian switching graph. In many existing works, time-invariant graphs are assumed. However, many physical graphs are often subject to abrupt variations in their structures, due to random link failures, sudden environmental disturbances, changing subsystem interconnections, etc. The Markovian switching (time-varying) topologies can be used to model some kind of changing graphs.
(2) About the steady state. It is noted that, based on sufficient off-line data, the distribution of graph connectivity can be established. The assumption that $r(t)$ is ergodic is a standard hypothesis, see, e.g., Hu and Hong (2007), Lou and Hong (2012) and Tang et al. (2012).
(3) About convergence and ahead of time. In this paper, our objective is to design distributed control laws to drive the followers to a point in the convex hull of the leaders as $t \to \infty$. Therefore, we focus on the asymptotic tracking errors. From this point of view, it is reasonable to assume that the graph starts from a stationary one.

4. Containment tracking with multiple dynamic leaders

In this section, we discuss the containment problem in two cases: one is that the leaders' velocities have special forms and the other is concerned with more general leaders' velocities.

4.1. A special case

We decompose the velocities $f_i(h,t)$ of the leaders as follows

$$f_i(h,t) = \alpha^{3/2}(t)\bar{f}_i(h,t), \tag{6}$$

where $\alpha(t) : [0,\infty) \to (0,\infty)$ is piecewise continuous satisfying $\int_{t_0}^{\infty}\alpha(s)ds = \infty$ and $\int_{t_0}^{\infty}\alpha^2(s)ds < \infty$.

We make the following assumption on the velocities of the leaders.

Assumption 1. There exists an unknown constant $M \geq 0$ such that

$$|\bar{f}_i(h,t)| \leq M, \quad \forall i \in \{n+1, \ldots, n+m\}. \tag{7}$$

Theorem 1. *Consider the leader–follower multi-agent system (2)–(3). Then, under Assumption 1 and the protocol (4) with $a(t) = \alpha(t)$, the containment can be solved in the asymptotic unbiased mean square sense if and only if the leader set $\mathcal{V}_l$ is globally reachable in the union digraph $\bigcup_{i=1}^{N}\bar{\mathcal{G}}_i$.*

More specifically, $\lim_{t\to\infty} E|x_i - h_{ci}|^2 = 0,\ i = 1, \ldots, n$.

Proof. *Necessity*: If the leader set $\mathcal{V}_l$ is not globally reachable in the union digraph $\bigcup_{i=1}^{N}\bar{\mathcal{G}}_i$, one can find some followers which are not connected to any leader. For these followers, by choosing different initial values from the leaders, one can easily prove the necessity.

Sufficiency: By Lemma 1, H is positive stable. Therefore, there exists a positive definite matrix P such that

$$PH^T + HP = 2I. \tag{8}$$

Let $\eta = x - H^{-1}B_l h(t)$, $\Lambda(t, r(t)) = (\eta(t), r(t))$, $\bar{f} = (\bar{f}_{n+1}, \ldots, \bar{f}_{n+m})$ and $V(t) = \eta^T P\eta$. For any $l > 0$, define the first exit time

$$\eta_l = \inf\{t : t \geq t_0,\ |\Lambda(t,p)| \geq l\}. \tag{9}$$

Let $t_l = \eta_l \wedge t$ for any $t \geq t_0$. Since $|\Lambda(\cdot)|$ is bounded in the interval $[t_0, t_l]$ a.s., $V(\Lambda)$ is bounded on $[t_0, t_l]$ a.s. Let $r(t) = p$, one has

$$\begin{aligned}\mathcal{L}V(\Lambda(t,p)) = {} & -\alpha(t)\rho(p)\eta^T(PH(p) + H^T(p)P)\eta \\ & - 2\alpha(t)\rho(p)\eta^T PH(p)H^{-1}B_l h \\ & + 2\alpha(t)\rho(p)\eta^T PB_l(p)h - 2\alpha^{3/2}(t)\eta^T PH^{-1}B_l\bar{f} \\ & + a^2(t)\rho^2(p)\,\mathrm{Tr}\left[G^T PG\right].\end{aligned} \tag{10}$$

By using Lemma 3 in Li, Liu, and Zhang (2012), one can get

$$\begin{aligned}& -2\alpha^{3/2}(t)\eta^T PH^{-1}B_l\bar{f}(h,t) \leq \frac{\alpha(t)}{\lambda_{\max}(P)}E(V(s)) \\ & \quad + M\sqrt{m}\alpha^2(t)\lambda_{\max}^2(P)\|H^{-1}B_l\|.\end{aligned} \tag{11}$$

Taking into consideration (10) and (11), in view of the Fatou lemma in Mao and Yuan (2006) and comparison theorem, we have

$$EV(t) \leq e^{-c_0\int_{t_0}^{t}\alpha(s)ds}EV(t_0) + C\int_{t_0}^{t} e^{-c_0\int_s^t \alpha(u)du}\alpha^2(s)ds. \tag{12}$$

Noting that $c_0 > 0$, by the definition of $\alpha(t)$, one has

$$\lim_{t\to\infty} E|\eta|^2 = 0. \tag{13}$$

Considering that

$$E|x_i - h_{ci}|^2 = E\left|x_i - \sum_{j=n+1}^{n+m} k_{ij}h_j\right|^2 \leq E|\eta|^2, \tag{14}$$

by using Lemma 2 and Definition 1, the containment can be solved in the asymptotic unbiased mean square sense.

When the leaders are stationary, by Theorem 1, we immediately get the following results.

Corollary 1. *Consider the leader–follower multi-agent system (2) – (3). Then, under the switching protocol (4), the containment can be achieved in the asymptotic unbiased mean square sense if and only if the leader set $\mathcal{V}_l$ is globally reachable in the union digraph $\bigcup_{i=1}^{N}\bar{\mathcal{G}}_i$.*

More specifically, $\lim_{t\to\infty} E|x_i - h_{ci}|^2 = 0\ i = 1, \ldots, n$.

Remark 4. This paper is the first to study the containment control with both Markovian switching topologies and stochastic noises in communication channels. When there is no stochastic noise, i.e. $G(r(t)) = 0$, Lou and Hong (2012) consider the distributed containment control problem of a multi-agent system guided by multiple leaders with random switching topologies, where some conditions on the tracking estimation are provided for the case where the target set determined by static and moving multiple leaders is convex. The topology condition of Lou and Hong (2012) is that the leader set is globally reachable in the union digraph. Although the communication channels are corrupted by white noises in this paper, the topology condition we require is the same as that in Lou and Hong (2012).

Remark 5. The existing results such as those in Lou and Hong (2012) solve the containment problem but cannot provide the specific locations the followers will eventually converge to. In this paper, we not only solve the containment tracking problem, but also establish that the ith followers will eventually converge to h_{ci}, which is nontrivial.

4.2. General case

For the general case, we adopt the assumption that the leaders' velocities are bounded, which is reasonable in the real world.

Assumption 2. There exists an unknown constant $M' \geq 0$ such that

$$|f_i(h,t)| \leq M', \quad \forall i \in \{n+1, \ldots, n+m\}. \tag{15}$$

Theorem 2. *Consider the leader–follower multi-agent system (2)–(3). Then, under Assumption 2 and the protocol (4) with $a(t) = 1$, the containment with bounded error can be achieved in the mean square sense if and only if the leader set $\mathcal{V}_l$ is globally reachable in the union digraph $\bigcup_{i=1}^{N} \bar{\mathcal{G}}_i$.*

Proof. *Necessity*: The necessity follows from Theorem 1.

Sufficiency: Similar to (12), one can get

$$EV(t) \leq e^{-c_0(t-t_0)} EV(t_0) + \frac{C}{c_0}(1 - e^{-c_0(t-t_0)}), \tag{16}$$

where $V(t)$ has the same definition with that in Theorem 1; $c_0 = \frac{1}{\lambda_{\max}(P)}$ and $C = M'\sqrt{m}\lambda_{\max}^2(P)\|H^{-1}B_l\| + \sum_{p=1}^{N} \rho(p) \operatorname{Tr}\left[G^T(p)PG(p)\right]$. By (16), one gets

$$\limsup_{t\to\infty} E|\eta|^2 \leq \frac{C}{c_0 \lambda_{\min}(P)}. \tag{17}$$

From (14) and (17), Definition 2 and Lemma 2, the result follows.

Remark 6. From (17) and the definitions of c_0 and C, when the leaders' velocities $f_i(h,t) = 0$ and the noise intensities $G_r = 0$, one has that $\lim_{t\to\infty} E|\eta|^2 = 0$. The containment can be solved in the asymptotic unbiased mean square sense.

Remark 7. From (16), for any $\varepsilon > 0$, by Chebychev's inequality,

$$P\{|\eta(t)| > \varepsilon\} \leq \frac{1}{\lambda_{\min}(P)\varepsilon^2}\left(EV(t_0) + \frac{C}{c_0}\right) \leq \varepsilon', \tag{18}$$

where ε' can be made small enough by choosing the parameter ε appropriately. Therefore, (18) approximates asymptotic tracking in probability in some sense.

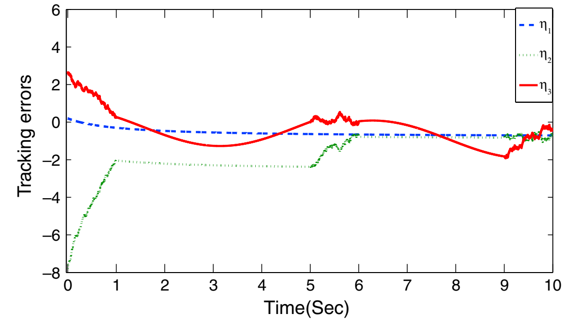

Fig. 1. The tracking errors between the followers and h_c.

5. A numerical simulation example

We consider a system consisting of two leaders and three followers. The leaders can be described by

$$\dot{h}_4(t) = \sin t, \qquad \dot{h}_5(t) = \frac{1}{(1+t)^2}.$$

One can get $h_c = (h_5, h_5, h_4)^T$. The Markov process $r(t)$ belongs to the space $S = \{1, 2\}$ with generator $\Gamma = (\gamma_{pq})_{2\times 2}$ given by $\gamma_{11} = -1, \gamma_{12} = 1, \gamma_{21} = 2$ and $\gamma_{22} = -2$. One gets $\pi_1 = \frac{2}{3}, \pi_2 = \frac{1}{3}$. The topology $\bar{\mathcal{G}}_1$ is described by $a_{21}(1) = b_{31}(1) = 1$ and $\bar{\mathcal{G}}_2$ is depicted as $b_{12}(1) = 1$, while other $a_{ij}(k)$s and $b_{is}(k)$s are zero, $i, j = 1, 2, 3,\ s, k = 1, 2$. The noise intensities are chosen as $\sigma_{21}(1) = \sigma_{15}(2) = \sigma_{34}(1) = 1$. With the initial conditions $x_1(0) = 9$, $x_2(0) = -8$, $x_3(0) = 3$, $h_4(0) = 0.3$, $h_5(0) = -0.2$, the tracking errors between the followers and h_c are given in Fig. 1, from which one can find that the containment has bounded errors.

6. Conclusions

In this paper, some necessary and sufficient conditions were provided for multi-agent containment tracking in the asymptotic unbiased mean square sense when the leaders' velocities have some special forms. For the case of more general leaders' velocities, some necessary and sufficient conditions have been given for the containment tracking with bounded errors in the mean square sense.

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